INSTITUTE OF DISTANCE AND OPEN LEARNING
GAUHATI UNIVERSITY
Home Assignment
M.A/M.Sc Mathematics
Semester - I (2013-2014)

GUIDELINES FOR SUBMISSION OF HOME ASSIGNMENTS:

1. Write your **NAME**, **ROLL NUMBER**, **SESSION**, **PAPER NUMBER**, **TOPIC SELECTED** and **EXAMINATION,** clearly on the top of the Front page of each paper.
2. Submit your Assignments **PAPER-WISE** Separately.
3. Each Paper carries a weightage of **16 marks**.
4. Keep a margin of about 1 inch on each side of the page.
5. **Stick File** not necessary.
6. **Copying** from others including **Xerox** from others strictly prohibited.
7. You can submit the essay written in your own hand-writing on **A-4** sized paper on **One Side** of each page **Only**.
8. Submit Your Assignments strictly on or before the due date as notified. Assignments received after the due date may not be considered for evaluation.
9. The last date of submission is ***25th October, 2013***.

N.B. Students are requested to follow the instructions strictly.

M101: Real Analysis and Lebesgue Measure (Answer any two) (8+8) Marks

1. Define uniform convergence at an interval. State and Prove Cauchy’s criterion for uniform convergence.
2. Show that a monotonic increasing function f in [a, b] is a function of bounded variation. Find the total variation of f.
3. Define the Lebesgue measurable set and state their properties.

   If E1, E2, ..., En be a finite sequence of disjoint measurable sets then prove that for any set A, $m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$.
4. Give the definition of Lebesgue measurable function. If f and g are measurable real valued
   functions on E, and c be a constant. Then prove each of the following functions is measurable on E:

   (a) f±c (b) cf (c) f±g

M102: Topology (Answer any two) (8+8) Marks

1. Define complete metric space. Establish the result
   Let (X,d) be a complete metric space, and let Y be a subset of X. Then Y is complete if and only if Y is closed.
2. Prove that if X is a second countable space, then any open base for X has a countable subclass which is also an open base.
3. Prove that a one to one continuous mapping of a compact space onto a Housdrauff space is a homeomorphism.

4. Show that if X is a Housdrauff space and has an open base whose sets are closed, then X is totally disconnected.

M103: Algebra (Answer any two) (8+8) Marks

1. Define solvable groups. Give two examples of solvable groups.
   Prove that if G/N and N are solvable and have solvable series of lengths l and m respectively, then G is solvable having a solvable subnormal series of length l+m.
2. Prove that a finite extension is algebraic. Is the converse true? Justify.
3. Discuss the Sylvester law of nullity.
4. Discuss the dual space of a vector space over a field F.

M104: Differential Equation (Answer any one) 16 Marks

1. State total differential equations. Discuss necessary and sufficient conditions for integrability of single differential equation Pdx+Qdy+Rdz=0.Obtain the solution of the differential equation
   $$(y^2z - y^3 + x^2y)dx - (x^2z + x^3 - xy^2)dy + (x^2y - xy^2)dz = 0 .$$

2. Define Power series, Analytic function, Ordinary and singular points in differential equation. Find the series solution near $x = 0$ of
   $(x + x^2 + x^3)y'' + 3x^2y' - 2y = 0$.

M105: Tensor and Mechanics (Answer any one) 16 Marks

1. State and prove Christoffels bracket of 1st and 2nd kind. Obtain covariant derivative of the covariant tensor of rank two.

2. Define Screw, Pitch and Wrench. To show that a given system of forces can be replaced by two forces equivalent to the given system in an infinite number of ways and that the tetrahedron formed by the two forces is of constant volume.

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